B Solutions to Problems

Solution to Problem 1. Recall the boundary condition and recursion from Lesson 10:

$$
\begin{aligned}
f(t) &= 0\\
f(i) &= \min_{j \text{ s.t. } (i,j)\in E} \{c_{ij} + f(j)\} \quad \text{for } i \in N \text{ s.t. } i \neq t
\end{aligned}
$$

where (N, E) is a directed graph, the target node is $t \in N$, the length of edge $(i, j) \in E$ is c_{ij}, and $f(i)$ is defined to be the length of a shortest path from node i to node t in the directed graph (N, E).

In the graph above, $t = 10$. So, starting with the boundary condition, we can find the shortest path lengths from nodes $1, \dots, 10$ to node 10 as follows:

$$
\begin{aligned}
f(10) &= 0\\
f(9) &= \min\{c_{9,10} + f(10)\} = \min\{1 + 0\} = 1\\
f(8) &= \min\{c_{8,10} + f(10)\} = \min\{1 + 0\} = 1\\
f(7) &= \min\{c_{7,10} + f(10)\} = \min\{1 + 0\} = 1\\
f(6) &= \min\{c_{6,8} + f(8), c_{6,9} + f(9)\} = \min\{4 + 1, 7 + 1\} = 5\\
f(5) &= \min\{c_{5,7} + f(7), c_{5,8} + f(8), c_{5,9} + f(9)\} = \min\{3 + 1, 9 + 1, 8 + 1\} = 4\\
f(4) &= \min\{c_{4,7} + f(7), c_{4,8} + f(8)\} = \min\{5 + 1, 2 + 1\} = 3\\
f(3) &= \min\{c_{3,5} + f(5), c_{3,6} + f(6)\} = \min\{4 + 4, 7 + 5\} = 8\\
f(2) &= \min\{c_{2,4} + f(4), c_{2,5} + f(5), c_{2,6} + f(6)\} = \min\{3 + 3, 9 + 4, 8 + 5\} = 6\\
f(1) &= \min\{c_{1,4} + f(4), c_{1,5} + f(5)\} = \min\{5 + 3, 2 + 4\} = 6
\end{aligned}
$$

Tracing through the recursion, we find that a shortest path from node 1 to node 10 is $(1, 5), (5, 7), (7, 10)$.